

Bellman Equation Handout: FrozenLake-v1

Random Policy $\pi(a|s) = \frac{1}{4}$ for all s, a | Discount $\gamma = 1$ | Slippery (stochastic transitions)

FrozenLake 4x4 State Map			
0: S	1: F	2: F	3: F
4: F	5: H	6: F	7: H
8: F	9: F	10: F	11: H
12: H	13: F	14: F	15: G

S = Start **F** = Frozen (safe) **H** = Hole (terminal, reward = 0) **G** = Goal (terminal, reward = 1)

Slippery ice: intended direction executed with prob $\frac{1}{3}$; each perpendicular direction also $\frac{1}{3}$

Actions: L = Left, D = Down, R = Right, U = Up

Reward: +1 only when entering the Goal state (state 15); 0 otherwise

Tree Diagram Legend (see next page)

○ White circle = **state node** — $V(s)$ value shown or to be computed

● Black circle = **action node** — $Q(s,a)$ value shown or to be computed

Blue edges = policy probability $\pi(a|s)$

Red edges = transition probability $P(s'|s,a)$ and reward r

... = branch continues with same pattern

Task: Compute the three blank values: $V(S_0)$, $Q(S_0,D)$, and $V(S_1)$

Bellman Expectation Equations:

$$V(s) = \sum_a \pi(a|s) \cdot Q(s,a)$$

$$Q(s,a) = \sum_{s'} P(s'|s,a) \cdot [r + \gamma \cdot V(s')]$$

Bellman Equation Tree: FrozenLake-v1, S_0 , $\pi(a|s) = \frac{1}{4}$, $\gamma = 1$

